

## SUBJECT TEACHING GUIDE

### G1902 - Measure Theory

#### Double Degree in Physics and Mathematics Degree in Mathematics

Academic year 2018-2019

1. IDENTIFYING DATA			
Degree	Double Degree in Physics and Mathematics Degree in Mathematics		Type and Year Optional. Year 5 Optional. Year 4
Faculty	Faculty of Sciences		
Discipline	Subject Area: Further Mathematical Analysis and Differential Equations Mention in Pure and Applied Mathematics		
Course unit title and code	G1902 - Measure Theory		
Number of ECTS credits allocated	6	Term	Semester based (2)
Web			
Language of instruction	English	Mode of delivery	Face-to-face

Department	DPTO. MATEMATICAS, ESTADISTICA Y COMPUTACION		
Name of lecturer	JESUS ARAUJO GOMEZ		
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Other lecturers			

### 3.1 LEARNING OUTCOMES

- To achieve a basic level on the various notions introduced in the course , and be able to do proofs of different results derived from those given in the classroom.
- To attain a level of abstraction allowing to put in the appropriate framework prior knowledge about integration theory.
- To be able to determine if a family of sets is a sigma-algebra and construct sigma-algebras generated by families of subsets. Identify complete and non-complete sigma-algebras.
- To compute integrals with respect to certain abstract measures . Check Riesz Theorem for positive functionals in some specific contexts.
- To establish a relation between different  $L_p$  spaces (both with respect to different  $p$  and different measures ). Calculate the norm in  $L_p$  of some particular functions, including the case  $p=\infty$ .

#### 4. OBJECTIVES

To develop a basic theory of abstract integration, both for positive-valued and complex-valued functions.

To develop the fundamental theorems on convergence and compare them with prior related results.

To know the most usual sigma-algebras and their completion.

To know the relation of positive linear functionals and their representation as integrals.

To know Radón-Nikodym Theorem

To study the relation between measurable and continuous functions.

To study the spaces  $L_p$  in the general case.

#### 6. COURSE ORGANIZATION

##### CONTENTS

1	<p>ABSTRACT INTEGRATION</p> <p>The concept of measurability.</p> <p>Borel sets.</p> <p>Simple functions.</p> <p>Elementary properties of measures.</p> <p>Completeness and regularity of measures.</p> <p>Arithmetic in <math>[0, \infty]</math>.</p> <p>Integration of positive functions.</p> <p>Integration of complex functions.</p> <p>Theorems of convergence.</p> <p>The role played by sets of measure zero.</p>
2	<p>POSITIVE BOREL MEASURES</p> <p>The Riesz Representation Theorem (positive measures)</p> <p>Regularity properties of Borel measures.</p> <p>Lebesgue measure.</p> <p>Continuity properties of measurable functions.</p> <p>Radon-nikodym Theorem</p> <p><math>L_p</math>-spaces</p>

#### 7. ASSESSMENT METHODS AND CRITERIA

Description	Type	Final Eval.	Reassessn	%
Midterm exam	Written exam	No	Yes	25,00
Midterm exam	Written exam	No	Yes	25,00
Final exam	Written exam	Yes	Yes	50,00
TOTAL				100,00
Observations				
<p>The continual assessment will consist of two exams covering each half of the course, in which the student will be asked to solve some problems and exercises. The final exam will cover all the contents seen in class along the semester. The final grade in June will be taken to be the maximum of the result achieved in the final exam and that obtained by a weighted combination where the final exam accounts for 50% and each midterm exam accounts for 25%. The final grade in September is determined by the grade achieved in the corresponding final exam. Use of notes or written material is not allowed during the exams.</p>				
Observations for part-time students				
Evaluation of part-time students will be conducted according to the norms given for full-time students				

## 8. BIBLIOGRAPHY AND TEACHING MATERIALS

### BASIC

W. Rudin. Analisis real y complejo (3 ed.), McGraw-Hill, 1988

M. Capinski, E. Kopp. Measure, Integral and Probability (2nd ed.) Springer 2004