SUBJECT TEACHING GUIDE

G92 - Commutative Algebra

Double Degree in Physics and Mathematics
Degree in Mathematics

Academic year 2020-2021

1. IDENTIFYING DATA					
Degree	Double Degree in Physics and Mathematics Degree in Mathematics			Type and Year	Compulsory. Year 4 Compulsory. Year 3
Faculty	Faculty of Sciences				
Discipline	Subject Area: Algebra Module: Compulsory Subjects				
Course unit title and code	G92 - Commutative Algebra				
Number of ECTS credits allocated	6	Term		Semester based (2)	
Web					
Language of instruction	Spanish	English Friendly	No	Mode of delivery	Face-to-face

Department	DPTO. MATEMATICAS, ESTADISTICA Y COMPUTACION
Name of lecturer	LUIS MIGUEL PARDO VASALLO
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Office	
Other lecturers	

3.1 LEARNING OUTCOMES
- Knowledge of notions and exemples of commutative rings, ideals and modules over these rings.
- Knowledge of basic properties of square matrices over elementary commutative rings: determinant, Hamilton-Cayley, Smith Normal form.
- Konwledge of statements and proofs about elementary properties of Euclidean domains, Principal Ideals Domains and Unique Factorisation Domains. Knowledge of examples coming from Number Theory and Algebraic, Diophantine and Differential Geometry.
- Knowledge of a proof of the existence of Smith Normal Form and its aplicaciones to classify finitely generated Abelian groups, Endomorphism Theory (Fronebius form) and solving of lenar Diophantine equations.
- Knowledge of the general statement and proof of the Chinese Remainder Theorem and its consequences : in Number Theory, classification of finteley generated Abelian groups (elementary divisors), and Linear Algebra (existencia of Jordan canonical forma and th change of basis matrix).
- Knowledge of some elementary proof of Hilbert-Kronecker's Nullstellensätz and its geometric meaning.
- Knowledge of basic properties of Noetherian rings and modules, including Hilbert's Basissatz. Knowledge of the statement and proof of Lasker-Noether Theorem about primary decomposition in Noetherian rings and modules.

4. OBJECTIVES
The main goal is to learn some basics in Commutative Algebra, as a supplement to previous formation in Algebra.
There is also an attempt to form students in writing and reading abstract mathematical notions, statements and proofs as a scientific language, with contents related to the general scope of Commutative Algebra and its applications.
Learn by experience the need of patience and effort to deeply understand mathematical methodology.

6. COURSE ORGANIZATION	
CONTENTS	
1	Notions of ring, ideal and module over a ring- Euclidean domains. Remainder Theorem. Structure of residue rings. Prime fields. R-algebra morphisms. theorems of Isomorphy. Abelian groups and modules. Endomorphism theory: annihilator ideal and minimal polynomial. Rings of polynomials in several variables and power series rings. Notions as the language of Geometry "à la Grothendieck". Notions as a basis for Algebraic Number Theory: quadratic extensions of Z. Rings and modules in other contexts: continuous functions, differential and analytic functions and their corresponding rings and modules.
2	Determinant. Group action on a set: orbits, transitive and faithful actions. Example: Cayley graphs. Symmetric groups and its generators: cycles of any order. Index as group morphism. Determinant of a matrix with coordinates in a commutative ring. Generalized Laplace Formula: the general linear group GL(n,R). Determinant as a group morphism. Unimodluar matrices. Hamilton-Cayley Theorem. Csanky-Leverrier-Fadeev-Souriau algorithm for computing characteristic polynomials..
3	Most elementary rings. Prime and maximal ideals. Zorn's Lemma and existence of maximal ideals. Zero divisors. Definition and presence in different contexts (continuous functions, differential functions, analytic functions). Euclidean domains. Bézout Identity with bounds. Euclid's algorithm. Euclid's algorithm in Z: Lamé's Theorem. Classic univariate Elimination theory: Sylvester matrix and resultant. principal ideal domains. Bézout's Identity. Existence of factorization in Principal Ideal Domains and Euclidean Domains. Unique factorization domains. Gauss Lemma and factorization in polynomial rings. Applications: RSA cryptosystem, factorization of univariate polynomials over finite fields (Berlekamp's method), error correcting codes and Hamming distance..
4	Torsion. Equivalence of finitely generated torsion free modules and finitely generated free modules over a principal ideal domain. Smith Normal Form: algorithms and existence. Solving linear Diophantine equations. First Structure Theorem on Finitely Generated Abelian Groups. Companion matrix of a polynomial and tensor of the residue ring. Frobenius form of an endomorphism and cyclic decomposition of K[X]-modules. Schwartz-Zippel Zero test: Fast algorithms to compute minimal polynomials of an endomorphism.
5	Chinese remainder Theorem. Applications: Secret Sharing, 2nd Structure Theorem for finitely generated Abelian Groups: elementary divisors. Modular Algorithms, Hadamard Inequality. Algebraically closed fields. Jordan forma of an endomorphism and CRT. Algorithms that compute the greatest common divisor of univariate polynomials in intrinsic dimension.
6	Nullstellensatz. Nilradical and Jacobson's radical. Zariski's topology. Algebraically closed fields (notion and main properties). Rings and ideals in Geometry. Ring of polynomial (also regular) functions (K[V]), ring of rational functions (K(V)), ideal of a variety (I(V)):Local and semi-local rings. Localization. An elementary proof of Hilbert's Nullstellensatz. Bézout Idenitty in K[X_1,..., X_n]. The language of categories: natural equivalence among categories. Examples in other contexts: Urysohn's Lemma and Tietze's Extension Theorem, Nullstellensatz of Banach-Stone-Cech, compactification through maximal spectra and Zariski topology.
7	Noetherian condition. Noetherian condition and partially ordered sets, assuming the Weak Election Axiom. Quqasi-compact topological spaces. Noetherian rings and modules. Hilbert's Basissatz. Zariski's topolgy is quasi-compact and consequence: irreducible components and primes. NAK's Lemma (Nakayama, Azumaya, Kronecker). Irreducible modules and primary submodules. Primary decomposition. Lasker- Noether existence Theorem of primary decomposition. Associated, support and annhilator: first Uniqueness theorem. The graph of the spectrum of a ring: Krul"s dimension. Noetherian condition and unique factorization (Nagat's Lemma). Introduction to standard and Groebner basis (Dixon's lemma and Buchberger algorithm).
8	Final exam

7. ASSESSMENT METHODS AND CRITERIA

Description	Type	Final Eval.	Reassessn	%
The student has to develop either some topic or some problemas and exercise. The value of this part ins 40 % of the final qualification.	Work	No	Yes	40,00
Several questions and problems related to contents of the course. Its value is 60% of the final qualification.	Written exam	Yes	Yes	60,00
TOTAL				100,00

Observations

In the Final examination, there will be some specific questions for those students that didn't pass continuous evaluation.

September call evaluation is 100% the results of the final examination.

Observations for part-time students

Part-time students will be evaluated by the same method described for full-time students.

8. BIBLIOGRAPHY AND TEACHING MATERIALS

BASIC

M.F. Atiyah, I.G. Macdonald, Introducción al álgebra conmutativa, ed. reverté 1973