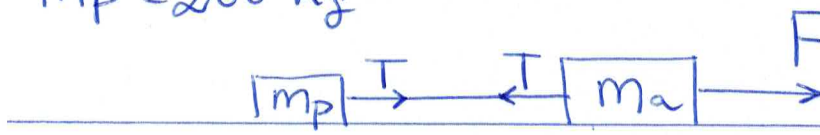
EJERCICIO 1

$m_a = 800\ kg$

$m_p = 200\ kg$

a)



Avioneta: $F - T = m_a a_2$

Planeador: $T = m_p a_2$

$\Big\} \Rightarrow F = (m_a + m_p)\, a_2$

$a_2 = \dfrac{F}{m_a + m_p}$: aceleración cuando remolca el planeador

Avioneta despegando sola: (MRUA), a_1

$v^2 = 2 a_1 \Delta x_1$ y $F = m_a a_1$

Avioneta despegando con planeador: (MRUA), a_2

$v^2 = 2 a_2 \Delta x_2 \Rightarrow a_1 \Delta x_1 = a_2 \Delta x_2$

$\Rightarrow a_2 (m_a + m_p) = a_1 m_a \Rightarrow \dfrac{a_2}{a_1} = \dfrac{m_a}{m_a + m_p} =$

$= \dfrac{\Delta x_1}{\Delta x_2} = \dfrac{800}{800+200} = 0.8 \Rightarrow \Delta x_2 = \dfrac{\Delta x_1}{0.8}$

$$\boxed{\Delta x_2 = \frac{180}{0.8} = 225\ m}$$

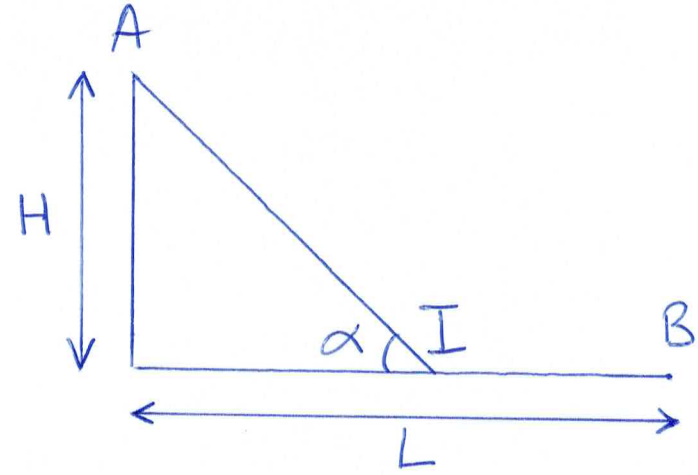
b) $a_2 = \dfrac{T}{m_p} = \dfrac{400}{200} = 2\ m/s^2 \Rightarrow a_1 = 2.5\ m/s^2$

$v^2 = 2 a_2 \Delta x_2 = 2 \cdot 2 \cdot 225 = 900 \Rightarrow \boxed{v = 30\ m/s}$

c) $a_3 = \dfrac{v^2}{2\Delta x_3} = \dfrac{F}{m_a + m_p'} = \dfrac{m_a a_1}{m_a + m_p'} = \dfrac{900}{2 \cdot 300} = 1.5\ \dfrac{m}{s^2}$

$\dfrac{800 \cdot 2.5}{800 + m_p'} = 1.5 \Rightarrow \boxed{m_p' = 533.33\ kg}$

EJERCICIO 2:



tramo AI: (MRUA)

$a = g \operatorname{sen}\alpha$

$d_{AI} = H/\operatorname{sen}\alpha = \frac{1}{2} a t_1^2$

$$\boxed{t_1 = \left(\frac{2H}{a \operatorname{sen}\alpha}\right)^{1/2} = \left(\frac{2H}{g \operatorname{sen}^2\alpha}\right)^{1/2}}$$

$v_I = (2gH)^{1/2}$, $d_{IB} = L - \frac{H}{\tan\alpha}$, $t_2 = \frac{d_{IB}}{v_I}$

$$\boxed{t_2 = \frac{L - \frac{H}{\tan\alpha}}{(2gH)^{1/2}}}$$

b) $$t = t_1 + t_2 = \left(\frac{2H}{g \operatorname{sen}^2\alpha}\right)^{1/2} + \frac{L - \frac{H}{\tan\alpha}}{(2gH)^{1/2}}$$

Si H, L y g son ctes. $\Rightarrow t = A + B \cdot (\operatorname{sen}\alpha)^{-1} + C \cdot (\tan\alpha)^{-1}$

con $A = \frac{L}{(2gH)^{1/2}}$, $B = \left(\frac{2H}{g}\right)^{1/2}$ y $C = -\left(\frac{H}{2g}\right)^{1/2}$

$$\frac{dt}{d\alpha} = -B \cdot (\operatorname{sen}\alpha)^{-2} \cos\alpha - C (\tan\alpha)^{-2} (\sec\alpha)^2 = 0$$

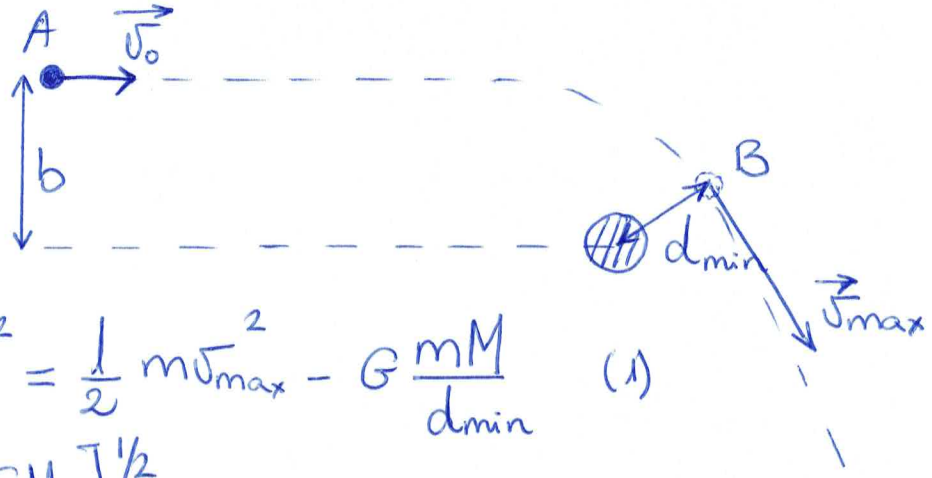
$$\frac{-B}{\tan\alpha \operatorname{sen}\alpha} - \frac{C}{\operatorname{sen}^2\alpha} = 0 \Rightarrow -B \cos\alpha = C$$

$$\cos\alpha = -C/B = \left(\frac{H}{2g}\right)^{1/2} \cdot \left(\frac{g}{2H}\right)^{1/2} = \frac{1}{2} \Rightarrow \boxed{\alpha_0 = 60^\circ}$$

c) $\tan\alpha_0 = \frac{H}{L_{min}} \Rightarrow$

$$\boxed{L_{min} = \frac{H}{\tan\alpha_0} = \frac{H}{\tan 60^\circ}}$$

EJERCICIO 3



$v_0 = 9000$ km/h

$b = 10^6$ Km

$$E_A = E_B \Rightarrow \frac{1}{2} m v_0^2 = \frac{1}{2} m v_{max}^2 - G\frac{mM}{d_{min}} \quad (1)$$

$$v_{max} = \left[v_0^2 + \frac{2GM}{d_{min}} \right]^{1/2}$$

$$\vec{L}_A = \vec{L}_B \Rightarrow L_A = L_B \Rightarrow m b v_0 = m d_{min} v_{max}$$

$$d_{min} = \frac{v_0}{v_{max}} b \Rightarrow v_{max} = \frac{b}{d_{min}} v_0$$

$$(2)\ \frac{1}{2} m v_0^2 = \frac{1}{2} m \left(\frac{b}{d_{min}}\right)^2 v_0^2 - G\frac{mM}{d_{min}} \quad (2)$$

Multiplicando (2) $\times \frac{2 d_{min}^2}{m v_0^2}$:

$$d_{min}^2 = b^2 - \frac{2GM}{v_0^2} d_{min} \Rightarrow d_{min}^2 + \frac{2GM}{v_0^2} d_{min} - b^2 = 0 \quad (3)$$

$$d_{min} = \frac{-\frac{2GM}{v_0^2} \pm \left[\left(\frac{2GM}{v_0^2}\right)^2 + 4b^2\right]^{1/2}}{2} = \begin{cases} 9.38 \times 10^8 \text{ m} \\ \end{cases}$$

$$\frac{2GM}{v_0^2} = \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{2500^2} = 1.28 \times 10^8 \text{ m}$$

$$\left(\frac{2GM}{v_0^2}\right)^2 = 1.64 \times 10^{16} \text{ m}^2 \Rightarrow \boxed{d_{min} = 9.38 \times 10^5 \text{ km}}$$

b) $v_{max} = \frac{b}{d_{min}} v_0 = 9594.7$ Km/h $= 2665.2$ m/s

c) En (3) $d_{min} = R_T$ y se despeja b:

$$b = \left(R_T^2 + 2\frac{GM}{v_0^2} R_T\right)^{1/2} = 2.93 \times 10^7 \text{ m} = 29300 \text{ km}$$

EJERCICIO 4

a) $Q_{1,0} = 4\pi\varepsilon_0 R_1 V_{1,0} = 10^{-7}\,C = 0.1\,\mu C$

b) $Q_{1,1} = 4\pi\varepsilon_0 R_1 V_{1,1}$ $\qquad V_{1,1} = V_{2,1}$

$Q_{2,1} = 4\pi\varepsilon_0 R_2 V_{2,1}$

$$Q_{1,1} + Q_{2,1} = 4\pi\varepsilon_0 V_{1,1}(R_1 + R_2) = Q_{1,0}$$

$$V_{1,1} = \frac{Q_{1,0}}{4\pi\varepsilon_0(R_1+R_2)} = V_{2,1} = 7500\ V$$

$$Q_{1,1} = 7.5\times10^{-8}\,C = 0.075\,\mu C$$

$$Q_{2,1} = 2.5\times10^{-8}\,C = 0.025\,\mu C$$

$$Q_{1,1} + Q_{2,1} = 0.1\,\mu C = Q_{1,0}$$

c) En cada descarga, se pierde $\frac{R_1}{R_1+R_2}$ de la carga en el paso anterior:

$$Q_{1,1} = 4\pi\varepsilon_0 R_1 V_{1,1} = 4\pi\varepsilon_0 R_1 \frac{Q_{1,0}}{4\pi\varepsilon_0(R_1+R_2)} = \frac{R_1}{R_1+R_2} Q_{1,0}$$

$$\Rightarrow \boxed{Q_{1,n} = \left(\frac{R_1}{R_1+R_2}\right)^n Q_{1,0}}$$

$$V_{1,n} = \frac{Q_{1,n}}{4\pi\varepsilon_0 R_1} \Rightarrow \boxed{V_{1,n} = \left(\frac{R_1}{R_1+R_2}\right)^n \frac{Q_{1,0}}{4\pi\varepsilon_0 R_1}}$$